ECON 110, Prof. Hogendorn

Problem Set 4

1. *Nineteen.* A firm's production function is $q = f(L) = 10 + L^{1/3}$. The wage of labor is \$10. The firm has a fixed cost of \$47,500.

   (a) What are this firm's total, marginal, average, and average variable cost curves? (Hint: as a general rule, don't expand expressions like $(a + b)^c$ unless you really have to!)

   (b) Suppose the firm is a perfect competitor and the price of the good is \$3,000. How much profit does the firm make? How much labor is employed?

   (c) If the price fell by 19%, what would be the percentage change in profits and employment at this firm? Graph what happens in two ways: on a graph of the marginal and average cost curves and on a graph of the production function.

   (d) After the price falls, should the firm shut down?

2. *EightFirms.* Suppose there are 8 firms supplying a given market. Each firm has the same total cost curve, which is

   $$TC(q) = 20 + 12q + 2q^2$$

   Each of the firms is a perfect competitor. *Market* demand is $q(p) = 60 - p$. What is the equilibrium price in this market? How much does each firm produce? Draw graphs to illustrate your answer.

3. *Nokia.* According to the *Wall Street Journal,* Nokia produces its cellphones using a very different production function than its competitors:

David Pringle, ``How Nokia Thrives by Breaking the Rules,'' January 3, 2003, pg. A7.

Salo, Finland -- THIS TOWN of 25,000 people on the northern fringe of Europe may not seem the obvious choice for a mobile-phone factory. Accessible only via a two-lane highway, Salo is a 90-minute drive from Helsinki's international airport. Average wages in the region are around 30 times those in low-cost manufacturing centers, such as China.

Yet the Nokia Corp. plant in Salo is the nerve center of the mobile-phone company's global-manufacturing operation. Rather than joining Dell Computer Corp., Cisco Systems Inc. and other big technology firms in contracting out manufacturing to low-cost specialists in Asia, Nokia makes most of its phones in its own factories, some of them in expensive locations, such as the U.S., Germany and this town in southern Finland.

Goldman Sachs estimates Nokia is making low-end handsets for as little as $70, matching the best of the Asian makers.

Nokia's operating margin on phones was 22% in the third quarter, and the company promised in December to top that in the fourth quarter.

Suppose the the price of a low-end cellphone is $85.40, the wage in China is $1 per hour and the wage in Finland is $30 per hour. We assume that workers in China and Finland are identical in terms of their own personal skills, trainability, etc.

(a) Suppose that Nokia's factory in Finland employs 650 workers and that a cellphone factory in China employs 10,000 workers. Both factories produce the 1,516 phones per hour. On the same graph, draw production functions $q(L)$ for both of

these factories (simply draw graphs, you don't have enough information to know the exact mathematical form of the production function). Assuming the firms are profit maximizers, what is the marginal product of labor at the Chinese firm? At Nokia's Finland plant?

(b) Which firm's labor demand curve is further to the right on a graph (with L on the x-axis and w on the y-axis)? Why?

(c) True or false: The fact that a *business* newspaper like the *Wall Street Journal* says that Nokia makes a profit margin of 22% shows that we cannot use the perfect competition model to think about Nokia because it does not set price equal to marginal cost. Draw a well-labeled cost-curve graph to justifies your answer.

4. *Generators.* It is 2 in the afternoon on a hot day in July. Everyone in the city has their air conditioning turned up, with the result that the typical household demands 6 kWh (kilowatt hours) of electricity during the 2-3pm time slot. Their demand is perfectly inelastic because they are so hot, they don't care about the price of electricity!

There are two generating companies (GenCos) serving this city. Each one operates an oil-fired power plant that can produce electricity according to the production function

$$f(g) = (540g)^{\frac{1}{3}}$$

where g is gallons of oil and q is kWh of electricity per household in the city. The price of oil is 200¢ per gallon. Each GenCo also has a fixed cost of 20¢ per household. The price of electricity is p and the GenCos are price-takers.

(a) The managers at GenCo A like to maximize their profits in terms of the quantity q of electricity they produce. Write down

the GenCo A profit function $\pi(q)$. Derive GenCo A's supply curve.

(b) The managers at GenCo B work differently. They figure out how much oil to buy to maximize profits. Write down the GenCo B profit function $\pi(g)$. Derive GenCo B's supply curve (this will require an extra step relative to your answer for part a).

(c) Describe in words why the two GenCos end up with the same supply curves.

(d) What is the market equilibrium price of electricity? Draw a graph of the market equilibrium.

(e) Derive and graph the marginal and average cost curves for one of the firms. At the market equilibrium price, calculate and label on the graph the profit or loss of the firm.

(f) Describe what will happen in this market in the long run, and show the effects in both the market graph and the graph of an individual firm. Just show how the curves will shift; don't calculate the actual quantities and prices in the long run.

Review Problems only, not to turn in:

5. *Low.* For the fisherman example from class, the cost curves are $MC(q) = 0.0512q$ and $AC(q) = \frac{50}{q} + 0.0256q$. Use the first derivative of AC to prove that MC crosses AC at the lowest point on the AC curve.

6. *MBAs.* The last recession was very hard on the strategic consulting industry. Firms like McKinsey, Bain, and Booz Allen & Hamilton laid off 30% of their workforce.

There were two components to the downturn. First, demand fell dramatically, in large part because of the demise of the dot-coms. Second, more executives began to have business school degrees

and/or experience with the consulting firms. This made the ``sage advice'' of the consultants themselves less useful and effectively reduced the marginal product of laborers with MBA (Master of Business Administration) degrees (see *The Economist*, 11/2/02, pg. 61).

For this problem, assume that the wage of MBAs is $100. (Note: for more realism, you can think of all money amounts in this problem in thousands.)

(a) Let a typical consulting firm have production function $f(L) = 10000L^{1/2}$ and the firm also incurs a fixed cost of 1000. What is this firm's total cost function, average cost function, average variable cost function, and marginal cost function?

(b) Graph these curves.

(c) If the price of consulting is $p = 2$ and there are 5 consulting firms, how many MBAs are hired?

(d) Suppose that p falls to 1.60 and also the production function changes to $f(L) = 10000L^{149/300}$. Now how many MBAs are hired?

7. *Returns.* Graph each of the following production functions and explain whether there are diminishing returns in each case? Consider a firm that has these production functions. What is the firm's demand curve for labor? (Let w be the wage and p be the price of the product.)

(a) $f(L) = 20L^{1/2}$

(b) $f(L) = 16002L - L^2$

8. *Revolution.* After Wesleyan, you take a job with McCoy consulting. It was a tough decision because McCoy's big rival, Barn & Co., was also recruiting you. And now the pressure is on because you are making a big presentation to Dolty, an auto parts manufacturer which is a perfectly competitive firm.

(a) The perfectly competitive price of a car bumper is \$500. Dolty uses steel to make bumpers according to the production function $f(S) = 1000S^{\frac{1}{12}}$ where S is tons of steel. The price of steel is \$800 per ton. What is Dolty's profit function $\Pi(q)$?

(b) Describe the condition for profit maximization that shows how many bumpers Dolty should produce.

(c) After you have shown the above, a team from Barn & Co. bursts into the room. Their young leader, known only by his initials A.G., says "Barn has a revolutionary new way to manage your firm. Don't think about bumpers, like these dinosaurs from McCoy! Instead, decide how much steel to buy!" He proceeds to write down a profit function $\Pi(S)$. Assuming he does this correctly, what does he write down? Show the condition for profit maximization using this function.

(d) Now it's up to you to save McCoy's reputation. Argue (in words) that the profit maximization condition for A.G.'s method is exactly the same as the profit maximization condition in your method, and that Barn & Co. has no revolutionary management technique.

Answers to Review Problems:

5. *Low_a.* At the lowest point on the AC curve, the slope is 0:

$$\frac{dAC}{dy} = -\frac{50}{y^2} + 0.0256 = 0 \Rightarrow y^2 = 1953.125 \Rightarrow y = 44.2$$

Setting MC=AC gives us

$$\frac{50}{y} + 0.0256y = 0.0512y \Rightarrow \frac{50}{y} = 0.0256y \Rightarrow y^2 = 1953.125 \Rightarrow y = 44.2$$
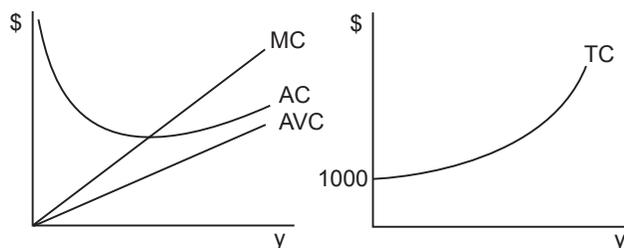
Either method gives the same answer.

6. *MBAs_a.*

   (a) Since $y = 10000L^{1/2}$, $L(y) = \left(\frac{y}{10000}\right)^2$. Thus,

$$
\begin{aligned}
TC(y) &= 1000 + wL = 1000 + 100\left(\frac{y}{10000}\right)^2 \\
AC(y) &= \frac{1000}{y} + \frac{y}{1000^2} \\
AVC(y) &= \frac{y}{1000^2} \\
MC(y) &= \frac{y}{500000}
\end{aligned}
$$

   (b)

   (c) We know that a profit-maximizing, perfectly competitive firm sets $p = MC(y)$. Here, that implies

$$\frac{y}{500000} = 2$$

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Solving this for y, we find that $y = 1,000,000$. Then $L(1,000,000) = 10,000$. Since there are 5 such firms, the total number hired is 50,000.

(d) Now the labor needed is:

$$L(y) = \left(\frac{y}{10000}\right)^{300/149}$$

and the optimal output solve:

$$MC(y) = 100\frac{1}{10000}^{300/149}\frac{300}{149}y^{151/149} = 1.60$$

Now the solution is $y = 750,000$ and $L(750,000) = 5,959$, for a total market employment of 29,795.

7. *Returns_a.*

(a) The graph is an increasing, concave function. $MP_L = 10L^{-1/2}$ which falls in L so there are diminishing returns. To find labor demand,

$$pMP_L = w \Rightarrow L^{-1/2} = \frac{w}{10p} \Rightarrow L(w) = \left(\frac{10p}{w}\right)^2$$

(b) This is also increasing concave, but it eventually reaches a maximum and then declines thereafter. This occurs at very large amounts of labor, however. $MP_L = 16002 - 2L$ also falls in L so there are diminishing returns. To find labor demand,

$$pMP_L = w \Rightarrow 16002p - 2Lp = w \Rightarrow 16002 - 2L = \frac{w}{p}$$

After we isolate L, we get

$$L(w) = \frac{16002 - \frac{w}{p}}{2}$$

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8. *Revolution_a.*

    (a) The profit function is total revenue minus total cost. To find the total cost, we need to know how much steel is used per bumper, which is just the inverse of the production function: $S(q) = (0.001q)^{12}$. Then:

$$\Pi(q) = 500q - 800(0.001q)^{12}$$

    (b) To maximize the profit function, take the derivative and set equal to 0:

$$\frac{d\Pi}{dq} = 500 - 12 \cdot 800(0.001q)^{11} \cdot 0.001 = 0$$

$$(0.001q)^{11} = \frac{500}{12 \cdot 800 \cdot 0.001} \Rightarrow q^* = 1432$$

In words, the condition is price (or marginal revenue) equals marginal cost.

    (c) AG's method also starts with profits equal total revenue minus total cost, but they are all measured in terms of S. Since the amount of bumpers produced is always $q = 1000S^{\frac{1}{12}}$, AG's profit function is:

$$\Pi(S) = 500 \cdot 1000S^{\frac{1}{12}} - 800S$$

He also takes the derivative and sets equal to 0:

$$\frac{d\Pi}{dS} = \frac{1}{12}500 \cdot 1000S^{\frac{-11}{12}} - 800 = 0 \Rightarrow S^{\frac{-11}{12}} = \frac{800 \cdot 12}{500 \cdot 1000} \Rightarrow S = 74.6$$

In words, this condition is price of bumpers times the marginal product of steel equals the price of steel. If we evaluate the production function at AG's optimal S, we get $q = 1000(74.6)^{\frac{1}{12}} = 1432$, the same answer as in part (b).

    (d) AG's method is identical. Because the production function provides a direct relationship between S and y, either decision making process yields the same result. McCoy's condition says that additional output should be produced until the

cost of another unit equals the revenue from selling it. Barn's condition says that additional steel should be purchased until the cost of the steel equals the revenue generated from selling bumpers made from the steel. These conditions are restatements of the same idea.