

ECON 110, Prof. Hogendorn

Problem Set 6

1. *OldGermans*. In Germany, the birth rate is low and the population is ageing. As a result, the working age population is falling at about 0.2% per year. It has been suggested that this population decline puts the German economy at risk. This question asks you to use our simple neoclassical model to evaluate that claim.

Let there be $\mathcal{L} = 243$ German workers who inelastically supply labor and who spend all of their income on beer consumption. These workers own the German beer firms which have aggregate production function $f(L) = 54L^{4/5}$. (Aggregate meaning we treat all the firms as if there were just 1.) Let $p = 1$.

- (a) Find the equilibrium real wage in the labor market and graph the labor market.
- (b) Verify that there is also equilibrium in the beer market and graph the production function. What share of workers' income comes from wages and what share from dividends?
- (c) Suppose that over 10 years, the German population falls and there are only $\mathcal{L}' = 198$ workers. Find the new general equilibrium.

2. *GrowingChina*. This problem discusses the Malthusian trap that has worried China for centuries and that the country now seems to have escaped. Let there be $\mathcal{L} = 1000$ Chinese workers who inelastically supply labor and who spend all of their income on rice. These workers own the Chinese rice firms which have aggregate production function $Y = f(L, K) = A(hL)^{2/3}K^{1/3}$. (Aggregate

meaning we treat all the firms as if there were just 1.) Let $A = 3.33$, $h = 1$, $p = 1$ and let $K = 729$. Note that the Chinese capital stock is constant until part (d) of this problem.

- (a) Find the equilibrium real wage and graph the labor market.
- (b) Verify that there is also equilibrium in the rice market and graph the production function. What is output per worker (Y/\mathcal{L})?
- (c) Suppose that over several years, the Chinese workforce rises to 1,728 workers. If nothing else changes, what is the new general equilibrium (the new wage and the new output per worker)? Why don't these new workers produce enough to keep the output per worker at least as high as before?
- (d) Consider the following changes to the production function: an increase in A , an increase in K , and an increase h . How would each of these help China escape the Malthusian trap? What is the name for each of these sources of growth?

3. *OldGermansSave*. As in *OldGermans*, there are 243 German workers who inelastically supply labor, but now they save 100 beers (in total) for the future and spend the rest of their income on beer consumption. These workers own the German beer firms which have aggregate production function $f(L) = \frac{54}{4}L^{4/5}$. The German beer firms have aggregate investment demand of $I = 1200/r$, where r is the real interest rate. Let $p = 1$.

- (a) Find the equilibrium real wage in the labor market and graph the labor market. Verify that there is also equilibrium in the beer market and graph the production function.
- (b) Graph the capital market. What is the equilibrium real interest rate?

- (c) What happens if Germans become more pessimistic and start saving 110 beers?
- (d) Go back to just 100 beers saved. What happens if the German government levies taxes of 60 beers but German Chancellor Angela Merkel drinks 80 beers?

Review Problem only, not to turn in:

4. *Uchitelle*. The following op-ed piece by Louis Uchitelle appeared in the New York Times on August 25, 2002. It provides some food for thought, but we can evaluate the arguments a lot more clearly in a simple macroeconomic model. At the end of the article is a suggestion for trying to model Uchitelle's idea.

In Alice in Wonderland fashion, we talk of expansion and ignore the contraction all around us. We convince ourselves that out of cost-cutting will come prosperity. But while cost-cutting can lift a single company or two, when practiced widely enough it can pull down an economy. And that is happening today.

Few economists acknowledge this dynamic. Corporate cost-cutting and labor-saving layoffs appear in the forecasts as the golden road to greater productivity and rising profits. Never mind that we have just fired the workers and extinguished the salaries that would have been spent on the merchandise and services to fatten the profits. With sales revenue failing to rise, we cut costs more. The process feeds on itself until there are not enough workers and salaries left to generate sales and profits.

There is hyperbole in this description, but not much. As a nation, we are caught in the strangest and perhaps most perilous recovery since the Depression featuring a

dynamic that William Dudley, chief domestic economist at Goldman Sachs, characterizes as "the corporate paradox of thrift."

"If everyone tries to cut costs and save more, no one saves more," he said. "If you and everyone else cut costs, costs do indeed go down, but revenue also goes down, so profits eventually go down, too. Collectively, we can't cut our way to prosperity."

Individual companies defending themselves, not the economy have good reason to throw themselves into this behavior. In one profit report after another this summer, the story has been the same: Sales revenue was flat or barely rose in the second quarter, but don't worry, profits were up. Cost-cutting and labor-saving efficiencies fattened the bottom line, and revenue will soon rise as lower costs allow us to cut prices and take sales away from competitors.

That is fine, for a while, for the winning company. But consider what happens in an imaginary country where Burger King and McDonald's are the entire business sector and the total national output 100 hamburgers a day, evenly divided between the companies matches the demand from this nation's consumers. Demand and sales revenue, however, stay flat. So Burger King lays off two workers and uses the saved wages partly to fatten profits and partly to discount prices by just enough to take sales and revenue away from McDonald's. And McDonald's responds in kind. But soon, the four laid-off workers, with little income, buy fewer hamburgers, and the nation's total consumption drops to 95 hamburgers a day. That sets off another round of cost-cutting and price discounting, and our imaginary nation sinks gradually

into stagnation or deep recession not unlike America in the 1930's.

Why isn't that danger uppermost in everyone's mind? Why are forecasters like James Glassman, a senior economist at J. P. Morgan Chase, so optimistic? In a nutshell, they expect an infusion of demand from somewhere that will reverse the cost-cutting and persuade companies to expand investment, production and hiring. Their main hopes are more tax cuts, more growth in federal spending and more interest rate cuts by the Federal Reserve. They also count on people to finance consumption by continuing to extract equity from their homes, which are still rising in value. Mainly, though, it is stimulus from Washington that for Mr. Glassman will save the day. "If Washington cannot get us moving toward full employment within a year," he said, "then there will be more federal stimulus. We have learned a lot since the Depression about how to fix the economy." BUT perhaps not enough. Perhaps we have become too accustomed to the other post-World War II recoveries, which were so different. There were no excesses to overcome from a stock market bubble and an insane rush by business to expand far beyond demand. Instead, when consumption rose, there was shortage and rising prices. To control inflation, the Fed pushed up interest rates. In response, consumption subsided, provoking a recession until rates came down and pent-up demand reasserted itself. Business stepped up investment to keep output abreast of demand, and recovery was on its way.

In the current cycle, however, consumption particularly for cars, housing and appliances never tapered off from very high levels during the nine-month recession

that started in January of last year. It has still not tapered off, but it is not rising, either, and that is a problem. Recovery requires increased consumption and business investment to make the economy grow. The danger today is that demand will decline instead, and recession will return or there will be prolonged stagnation. Unfortunately, Mr. Dudley's "corporate paradox of thrift" is pushing hard in that direction.

Let there be 32 workers who inelastically supply labor and who spend all of their income on hamburgers. Let McDonald's and Burger King be identical firms that initially have production function $f(L) = 25L^{0.25}$. Let $p = 1$. For each of the following, illustrate with graphs of the labor market and the hamburger production function, and also show your answers mathematically.

- (a) Find the equilibrium real wage in the labor market. Verify that there is also equilibrium in the hamburger market. (Remember there are two identical firms.)
 - (b) One way of interpreting Uchitelle's notion of "cost cutting" is that the firms' production functions change to $f(L) = 25L^{0.28}$. Find the new general equilibrium. Is this consistent with Uchitell's story?
 - (c) Suppose instead that the production function changed to $f(L) = 25(L + 2)^{0.25}$. Does this labor-saving technology have the same impact on real wages as in (b)? Why or why not?
5. *Botswana*. Let's consider the IMF's policy toward Botswana (criticized by Joseph Stiglitz in his book *Globalization and its Discontents*, pg. 38). Normalize Botswana's working population to $\mathcal{L} = 100$. Let Botswana initially have a production function $y = f(L) = 380L^{1/2}$, and assume the firms represented by this function are owned by the workers. Set the price of y equal to 1.

- (a) Find the equilibrium real wage in Botswana's labor market and graph the labor market. Also graph the production function.
- (b) Private Botswanans were saving 700. The government was spending 25% of GDP and collecting taxes of 25% of GDP. Firms' investment demand function was $I = \frac{7630}{r}$. We will ignore foreign capital flows (actually, they were considerable in real life). Graph the capital market, showing the private, government, and total savings curves and the investment curve. What was the real interest rate?
- (c) Stiglitz claims that Botswana faced two negative shocks in 1981 due to drought and problems in the diamond industry. We'll model this by saying that the production function changed for the worse to $y = f(L) = 350L^{1/2}$. What was the new real wage and the new output?
- (d) The government of Botswana collected taxes of 25% of the new, lower real GDP, but it did not take the IMF's advice and continued to spend the same real amount as before the shocks. Real private saving and investment demand remained unchanged. Graph what happened in the capital market and find the new real interest rate.
- (e) The neoclassical model suggests that real wages in Botswana will grow more slowly as a result of the government's decision in (c). Why? What is an argument against this view?

Answers to Review Problems:

4. *Uchitelle_a.*

- (a) The profit of one of the firms is $\pi(L) = p \cdot 25L^{1/4} - wL$. The first order condition for the optimal L to demand is

$$\frac{d\pi}{dL} = 6.25L^{-3/4} - w = 0$$

Solving for L , we find that the firm's labor demand is

$$L^D = 11.5w^{-4/3}$$

Setting labor supply equal to *market* labor demand gives us:

$$32 = 23w^{-4/3} \Rightarrow w^* = 0.78$$

At this wage, each firm hires $L^D = 16$ workers and produces an output of $f(16) = 50$ hamburgers. Each firm makes a profit of $\pi(16) = 50 - 0.78 \cdot 16 = 37.5$.

The income of the consumers is the total wage bill of $0.78 \cdot 32 = 25$ plus the dividends earned from owning the firms, for a total of $25 + 2 \cdot 37.5 = 100$. With nothing else to buy, this means consumers demand 100 hamburgers, which is the total output of the firms.

(b) We repeat the above steps for the new production function:

$$1 \cdot MP_L = 7L^{-0.72} = w$$

Solving for L , we find that the firm's labor demand is

$$L^D = 14.9w^{-1.39}$$

Setting labor supply equal to *market* labor demand gives us:

$$32 = 29.8w^{-1.39} \Rightarrow w^* = 0.95$$

At this wage, each firm hires $L^D = 16$ workers and produces an output of $f(16) = 54.3$ hamburgers. Each firm makes a profit of $\pi(16) = 54.3 - 0.95 \cdot 16 = 39.1$.

The income of the consumers is the total wage bill of $0.95 \cdot 32 = 30.4$ plus the dividends earned from owning the firms, for a total of $30.4 + 2 \cdot 39.1 = 108.6$. With nothing else to buy, this means consumers demand 108.6 hamburgers, which is the total output of the firms.

(c) Now we have

$$MP_L = 6.25(L + 2)^{-3/4} = w$$

Solving for L , we find that the firm's new labor demand is

$$L^D = 11.5w^{-4/3} - 2$$

Setting labor supply equal to *market* labor demand gives us:

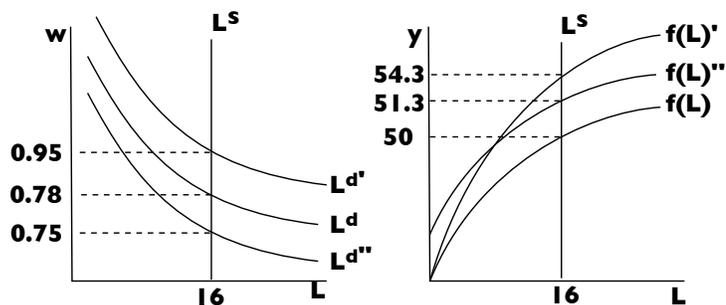
$$32 = 23w^{-4/3} - 4 \Rightarrow w^* = 0.75$$

At this wage, each firm hires $L^D = 16$ workers and produces an output of $f(16) = 51.5$ hamburgers. Each firm makes a profit of $\pi(16) = 51.5 - 0.75 \cdot 16 = 39.5$.

The income of the consumers is the total wage bill of $0.75 \cdot 32 = 24$ plus the dividends earned from owning the firms, for a total of $24 + 2 \cdot 39.5 = 103$. With nothing else to buy, this means consumers demand 103 hamburgers, which is the total output of the firms.

In a sense, this is consistent with Uchitelle's story because the real wage falls and firms profits increase. But since the profits go back to the workers, this is actually a good thing. One can easily see, however, how this could lead to greater income inequality in a slightly more complicated model.

This change in the production function actually decreases df/dL , the marginal product of labor. It increases total product, so the economy grows, but the contribution of the last worker hired actually falls. As a result, demand for labor decreases, and so does the real wage. In part (b), MP_L increased, thus increasing the real wage.



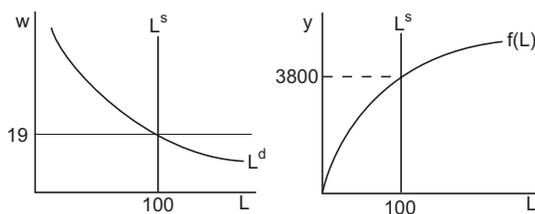
5. Botswana_a.

- (a) Labor demand can be found quickly by remembering that $pMP_L = w$, so

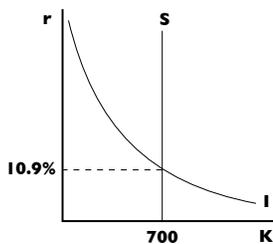
$$190L^{-1/2} = w \Rightarrow L^{1/2} = \frac{190}{w} \Rightarrow L^D(w) = \frac{36100}{w^2}$$

Then setting labor demand equal to labor supply gives us:

$$L^D(w) = L^S \Rightarrow \frac{36100}{w^2} = 100 \Rightarrow w = 19$$



- (b) Government spending and taxes are the same, so government saving is zero. Thus, capital market equilibrium occurs where private saving, S^P equals I : $\frac{7630}{r} = 700 \Rightarrow r = 10.9\%$



(c) We have to find the new labor curve:

$$175L^{-1/2} = w \Rightarrow L^{1/2} = \frac{175}{w} \Rightarrow L^D(w) = \frac{30625}{w^2}$$

Then setting labor demand equal to labor supply gives us:

$$L^D(w) = L^S \Rightarrow \frac{30625}{w^2} = 100 \Rightarrow w = 17.5$$

So real wages fall to 17.5 and output falls to $f(100) = 3500$.

(d) The government is still spending $25\% \cdot 3800 = 950$ but is only collecting taxes of $25\% \cdot 3500 = 875$. Thus it is running a deficit of 75, which raises the equilibrium real interest rate to 22%.

(On a graph, there should be a vertical line for government saving at -75 and a vertical line for total saving at 625. The investment demand is the same.)

(e) The neoclassical model says that the lower private investment will lead to less capital deepening and therefore slower economic growth. A counter-argument is that in the midst of its problems, government spending is needed to stabilize education, healthcare, and so forth, and that these provide more improvements to the production function than private capital.