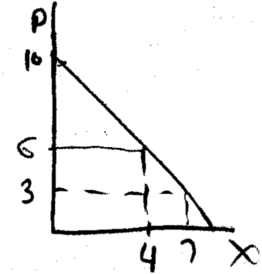
① $(p_1+t)x_1+(p_2-20)x_2 = m+600$

② If $x = 10 - p$, then $\bar{p} = 10$.



CS when $p = 6$ is $\frac{1}{2}(10-6)(10-6)$

$= \frac{1}{2}(4)(4) = 8$

CS when $p = 3$ is $\frac{1}{2}(10-3)(10-3)$

$= \frac{1}{2}(7)(7) = 24.5$

$\Delta CS = 24.5 - 8 = 16.5$

③ (a) $\mathcal{L} = \frac{RX}{R+X} - \lambda(p_R R + X - M)$

$\max_{R,X,\lambda} \mathcal{L}$ leads to first order conditions:

$$\frac{\partial \mathcal{L}}{\partial R} = \frac{X}{R+X} - \frac{RX}{(R+X)^2} - \lambda p_R = 0 \Rightarrow \lambda = \frac{X}{R+X}\frac{1}{p_R} - \frac{RX}{(R+X)^2}\frac{1}{p_R}$$

$$\frac{\partial \mathcal{L}}{\partial X} = \frac{R}{R+X} - \frac{RX}{(R+X)^2} - \lambda = 0 \Rightarrow \lambda = \frac{R}{R+X} - \frac{RX}{(R+X)^2}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -(p_R R + X - M) = 0$$

$$\lambda = \lambda \Rightarrow \frac{X}{R+X}\frac{1}{p_R} - \frac{RX}{(R+X)^2}\frac{1}{p_R} = \frac{R}{R+X} - \frac{RX}{(R+X)^2}$$

$$\Rightarrow X\frac{1}{p_R} - \frac{RX}{R+X}\frac{1}{p_R} = R - \frac{RX}{R+X}$$

$$\Rightarrow RX + X^2 - RX = p_R(R^2 + RX - RX)$$

$$\Rightarrow X^2 = p_R R^2$$

$$\Rightarrow R = \frac{1}{\sqrt{p_R}} X$$

From budget constraint, $X = M - p_R R$. Sub this in:

$$R = \frac{M - p_R R}{\sqrt{p_R}} \Rightarrow R\left(1 + \frac{p_R}{\sqrt{p_R}}\right) = \frac{M}{\sqrt{p_r}}$$

$$\Rightarrow R\left(\frac{\sqrt{p_R} + p_R}{\sqrt{p_R}}\right) = \frac{M}{\sqrt{p_R}} \Rightarrow R = \frac{M}{p_R + \sqrt{p_R}}$$ ✓

(b) $R = \frac{1000}{64+8} = 13.9$ $\quad p_R R = 64 \cdot 13.9 = \$889.60$ $\quad u = \frac{RX}{R+X} = \frac{1534.6}{124.3} = 12.3$

$X = 1000 - 64R = 110.40$

(c) $R = \frac{1000-850}{4+2} = 25$ $\quad p_R R + 850 = 4 \cdot 25 + 850 = 950$ $\quad u = \frac{RX}{R+X} = \frac{1{,}250}{75} = 16.7$

$X = 1000 - 850 - 4R = 50$

4

(a) $MRS = -\frac{p_C}{p_H}$

$$-\frac{\frac{\partial u}{\partial C}}{\frac{\partial u}{\partial H}} = -\frac{aC^{a-1}H^{1-a}}{(1-a)C^aH^{-a}} = -\frac{a}{1-a}\frac{H}{C} = -\frac{p_C}{p_H}$$

$$\Rightarrow -\frac{a}{1-a}\frac{15}{55} = -\frac{1}{3} \Rightarrow 3\cdot 15a = 55(1-a)$$

$$\Rightarrow 45a = 55 - 55a \Rightarrow 100a = 55 \Rightarrow a = \frac{55}{100} = \frac{11}{20}$$

(b) From (a), $\frac{a}{1-a}\frac{H}{C} = \frac{p_C}{p_H} \Rightarrow \frac{\frac{11}{20}}{\frac{9}{20}}\frac{H}{C} = \frac{p_C}{p_H} \Rightarrow 11H = 9\frac{p_C}{p_H}C$

From budget constraint, $p_C C + p_H H = m$. So $p_C C = m - p_H H$.

Sub together to get $11H = 9\frac{m - p_H H}{p_H} = \frac{9m}{p_H} - 9H$

$$\Rightarrow 20H = \frac{9m}{p_H} \Rightarrow H = \frac{\frac{9}{20}m}{p_H}$$

And $p_C C = m - p_H H = m - p_H\frac{\frac{9}{20}m}{p_H}$

$$\Rightarrow C = \frac{\frac{11}{20}m}{p_C}$$

So at $p_H = 2$, $H = \frac{\frac{9}{20}100}{2} = 22.5$, $C = 55$

(c) Initial bundle was $C = 55$, $H = 15$.

Slutsky compensated demand is:

$$H(p_H, p_C, p_C 55 + p_H 15) = \frac{\frac{9}{20}(p_C 55 + p_H 15)}{p_H}$$

At $p_H = 2$, $p_C = 1$, Slutsky comp. demand is:

$$H(2, 1, 55 + 2\times 15) = \frac{\frac{9}{20}(55+30)}{2} = \frac{9\cdot 85}{40} = \frac{9\cdot 17}{8} = 19.1$$

So the Slutsky substitution effect is: $19.1 - 15 = 4.1$