(1) True. Since $L = 100$ is fixed, we need $\lim_{F \to \infty} f(L,F) = \infty$.

This is only true if $MP_F > 0$ for all F. Actually, $\frac{dMP_F}{dF} > 0$ is a sufficient condition but not necessary.

For example, if $f(L,F) = L^{.2}F^{.2}$, $MP_F = .2L^{.2}F^{-.8}$ which is greater than 0 for all F even though $\frac{dMP_F}{dF} < 0$.

(Grade of 4 for a good discussion of situation, 5 for stating that condition is too strong and/or mentioning that $MP_F < 0$ would be required to make $\lim_{F \to \infty} f(L,F) < \infty$.)

(2) Yes, if $ATC > p > AVC$, the firm loses money but would lose even more money if it shut down because then none of the fixed cost would be covered.

③ (a) $c(12{,}030|K) = 1976.81$ (+ 250 if you include, as I should have, the cost of capital)

$$AC(y|K) = \frac{c(y|K)}{y} = \frac{3}{y}\left(\frac{\sqrt{34500+28y}-250}{14}\right)^2 \quad \left(+\frac{250}{y}\right)$$

$$MC(y|K) = \frac{d}{dy}c(y|K) = 3\cdot 2\left(\frac{\sqrt{34500+28y}-250}{14}\right)\left(\frac{1}{14}\frac{1}{2}(34500+28y)^{-\frac{1}{2}}\right)28$$

$$= 6\left(\frac{\sqrt{34500+28y}-250}{14}\right)\frac{1}{28\sqrt{34500+28y}}\,28$$

$$= 6\left(\frac{\sqrt{34500+28y}-250}{14}\right)\frac{1}{\sqrt{34500+28y}}$$

(b) $\min\ 3L+10K$ s.t. $50\sqrt{LK}+40K+7L=y$ $\Rightarrow$ $\min \mathcal{L} = 3L+10K-\lambda\left(50\sqrt{LK}+40K+7L-y\right)$

FOCs: $\frac{\partial\mathcal{L}}{\partial L} = 3-\lambda 50\sqrt{K}\frac{1}{2}\frac{1}{\sqrt{L}}-\lambda 7 = 0 \qquad \lambda = \frac{3}{\frac{50}{2}\frac{\sqrt{K}}{\sqrt{L}}+7}$

$\frac{\partial\mathcal{L}}{\partial K} = 10-\lambda 50\sqrt{L}\frac{1}{2}\frac{1}{\sqrt{K}}-\lambda 40 = 0 \qquad \lambda = \frac{10}{\frac{50}{2}\frac{\sqrt{L}}{\sqrt{K}}+40}$

$\frac{\partial\mathcal{L}}{\partial\lambda} = 50\sqrt{LK}+40K+7L-y=0$

$\lambda=\lambda \Rightarrow \frac{3}{25\frac{\sqrt{K}}{\sqrt{L}}+7} = \frac{10}{25\frac{\sqrt{L}}{\sqrt{K}}+40} \Rightarrow 75\frac{\sqrt{L}}{\sqrt{K}}+120 = 250\frac{\sqrt{K}}{\sqrt{L}}+70$

$\Rightarrow 250\frac{\sqrt{K}}{\sqrt{L}} - 75\frac{\sqrt{L}}{\sqrt{K}} = 50$

$\Rightarrow 250^2\frac{K}{L} - 2\cdot 250\cdot 75 + 75^2\frac{L}{K} = 250$

(4) (a)

$$X(p) = S((1-g)p) \qquad X(p^*) = A p^{*\varepsilon}$$

$$A p^{\varepsilon} = d(1-g)p \qquad = A\left(\frac{d(1-g)}{A}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$A p^{\varepsilon-1} = d(1-g)$$

$$p^{\varepsilon-1} = \frac{d(1-g)}{A} \qquad = A^{1-\frac{\varepsilon}{\varepsilon-1}}\, d^{\frac{\varepsilon}{\varepsilon-1}}\,(1-g)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$p^* = \left(\frac{d(1-g)}{A}\right)^{\frac{1}{\varepsilon-1}} \qquad = A^{-\frac{1}{\varepsilon-1}}\, d^{\frac{\varepsilon}{\varepsilon-1}}\,(1-g)^{\frac{\varepsilon}{\varepsilon-1}}$$

(b) $g p^* X(p^*)$

$$= g\left(\frac{d(1-g)}{A}\right)^{\frac{1}{\varepsilon-1}} A^{-\frac{1}{\varepsilon-1}}\, d^{\frac{\varepsilon}{\varepsilon-1}}\,(1-g)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= g A^{-\frac{1}{\varepsilon-1}-\frac{1}{\varepsilon-1}}\, d^{\frac{1}{\varepsilon-1}+\frac{\varepsilon}{\varepsilon-1}}\,(1-g)^{\frac{1}{\varepsilon-1}+\frac{\varepsilon}{\varepsilon-1}}$$

$$= g A^{-\frac{2}{\varepsilon-1}}\, d^{\frac{\varepsilon+1}{\varepsilon-1}}\,(1-g)^{\frac{\varepsilon+1}{\varepsilon-1}}$$

(c) $$\frac{d\,\text{revenue}}{dg} = \underbrace{A^{-\frac{2}{\varepsilon-1}}}_{+}\;\underbrace{d^{\frac{\varepsilon+1}{\varepsilon-1}}}_{+}\left[-\frac{\varepsilon+1}{\varepsilon-1}(1-g)^{\frac{\varepsilon+1}{\varepsilon-1}-1}g + (1-g)^{\frac{\varepsilon+1}{\varepsilon-1}}\right]$$

$$\text{sign}\left(\frac{d\,\text{rev.}}{dg}\right) = \text{sign}\left((1-g)^{\frac{\varepsilon+1}{\varepsilon-1}} - \frac{\varepsilon+1}{\varepsilon-1}(1-g)^{\frac{\varepsilon+1}{\varepsilon-1}-1}g\right)$$

$$= \text{sign}\left(1 - \frac{\varepsilon+1}{\varepsilon-1}(1-g)^{-1}g\right)$$

Thus, $\frac{d\,\text{rev.}}{dg} < 0$ if $1 - \frac{\varepsilon+1}{\varepsilon-1}\frac{g}{1-g} < 0$

$$\Rightarrow \quad \frac{\varepsilon+1}{\varepsilon-1}\frac{g}{1-g} > 1$$

$$\Rightarrow \quad (\varepsilon+1)g < (\varepsilon-1)(1-g)$$

$$\Rightarrow \quad \varepsilon g + \cancel{g} < \varepsilon - 1 - \varepsilon g + \cancel{g}$$

$$\Rightarrow \quad 2\varepsilon g < \varepsilon - 1$$

$$\Rightarrow \quad 2g > 1 - \frac{1}{\varepsilon}$$

$$\Rightarrow \quad 2g - 1 > -\frac{1}{\varepsilon}$$

$$\Rightarrow \quad \varepsilon(2g-1) < -1$$

$$\Rightarrow \quad \varepsilon(1-2g) > 1$$

$$\Rightarrow \quad \varepsilon < \frac{1}{1-2g} \quad \text{if } g > .5$$

Grade of 4 if a previous math error made the signing part significantly easier.
Grade of 5 even with a math error if final part is close to correct.
These changes needed to preserve fairness for those who got (a), (b), and first part of (c) right.