

ECON 201, Prof. Hogendorn: Problem Set #3

1. Let your utility function be

$$u(G, V) = G^{0.9}V^{0.1}$$

where G is ounces of gin and V is ounces of vermouth. Let the price of gin be \$1 per ounce and the price of gin be \$0.40 per ounce.

- (a) Using only the information above, describe what proportions you use to make a martini.
 - (b) Now suppose that you have \$4 available to spend on martinis. Use the Lagrangian to solve the utility maximization problem, and then show how many ounces of martinis you drink.
 - (c) If you had one more dollar, how much additional utility would you receive?
2. One good you could buy is an Audi TT Roadster, which would cost you \$33,000. Suppose that your wealth is currently valued at \$66,000 (this could be a value placed on your expected income over the next several years). The implication is that you could in fact buy two Audis. Nevertheless, you probably would say "It's too expensive." This question asks you to explain what that means according to formal economic theory.
- (a) Write down your budget line in terms of Audis, A , and the numeraire good, X . Draw this on a graph.
 - (b) Suppose you have utility function

$$u(A, X) = A^c + X^{1/2}$$

and suppose you would not buy one Audi (although you might buy a fraction of an Audi if that were possible). Graph this situation and show what values of c would describe it. (I.e. your answer should be something like $c > 23$ or $c < 0.2$.)

3. Suppose that a typical college student's utility for Napster songs, S , and other goods, X , can be given by

$$u(S, X) = \sqrt{SX}$$

- (a) Use the Lagrangian to solve for the utility maximizing number of songs to download from Napster. Let the amount of money available be M and let the price of downloading a song be P_S . The price of other goods is \$1.
- (b) Suppose that in the past, downloading a song from Napster cost \$0.25 (the money cost was zero, but there was still an opportunity cost of the time it took to download a song). What was the typical student's utility? How many songs were downloaded?

- (c) One possible new pricing plan for Napster is to charge \$1.05 per song to cover copyright fees. Adding in the time cost, this would make $P_S = \$1.30$. If $M = \$200$, What would be the student's utility under this plan? How many songs would be downloaded? How much would the total copyright fees be? (Remember, the copyright fee is only \$1.05 per song.)
- (d) Alternatively, Napster could charge a subscription to cover copyright fees. Suppose the subscription were \$100. Songs could be downloaded for free, so adding in the time cost, $P_S = \$0.25$. What would be the student's utility under this plan? How many songs would be downloaded? How much would the total copyright fees be?
4. Congress is allocating \$1 million of research funding. It can fund research in surfboard safety, B , or in snowmobile safety, S . Since research is measured in dollars, the price of each good is \$1.

The utility of each Californian is

$$u_c(B, S) = \sqrt{9B + S}$$

The utility of all other Americans is

$$u_a(B, S) = \sqrt{4B + 6S}$$

- (a) To make this decision, Congress uses the social welfare function

$$W(u_a, u_c) = 0.9u_a + 0.1u_c$$

What is the optimal allocation?

- (b) Suppose that Congress agrees to a compromise: the Rawlsian social welfare function will be used, so that research will be allocated to maximize

$$W(u_a, u_c) = \min\{u_a, u_c\}$$

What is the optimal allocation? Hint: first review Varian Chapter 4, problem 4, pg. 70 and Figure 5.5 on pg. 79; then find the regions of (B, S) space where Californians and other American do best; then maximize the utility of the worse-off group in each of these regions.