ECON 201, Prof. Hogendorn: Problem Set #8

1. Suppose the long run total cost curve for firms in an industry is

$$C(y) = \frac{1}{3}y^2 + 100$$

The demand curve is $X(p) = 10000 - 3p$.

   (a) What is the long-run supply curve of an individual firm?

   (b) How much will each firm produce in long run perfectly competitive equilibrium?

   (c) How many firms will be in the market in long run perfectly competitive equilibrium?

2. Up to 1998, Union Pacific (UP) and Burlington Northern Santa FE (BNSF) jointly used a railroad line from Dawes, TX to Avondale, LA. Control of the line was complicated, but we can approximate the arrangement by saying that UP had 80% control of the Beaumont Segment (B) of the line from Dawes to Iowa Junction, LA and 20% control of the Avondale Segment (A) from Iowa Junction to Avondale. BNSF had the remaining shares of control.

   The two railroads are essentially identical, so we can assume that both have the same production function over this route:

$$f(X_B, X_A) = X_B^{0.7} X_A^{0.7}$$

   (a) Calculate the MRTS of X_B for X_A for both railroads at the endowment point. (Note, 80% ownership of the Beaumont segment implies $\omega_B = 0.8$.)

   (b) Draw an Edgeworth box showing the endowment and isoquants for the firms. (The isoquants do not have to be plotted to match the production function perfectly.)

   (c) In 1998, the two railroads agreed to change the ownership structure. Assume they traded at a market price as price-takers, and assume both seek to maximize output. If we set X_A as the numeraire, what was the price of X_B? What was the final allocation of X_B and X_A?

   (d) Show the trading in your diagram.

3. Let the wage of Burger King workers be \$5 per hour. Let the price of a Whopper be p. Let the production function be

$$f(L, B) = (L^{0.5} + 2B^{0.5})^2$$

   output is measured in Whoppers per hour, L is the number of workers working during the hour, and B is the number or broilers.

(a) Find the number of workers hired as a function of B and p. (Assuming that Burger King believes it cannot change the price of Whoppers.)

(b) Use derivatives to show the change in workers if one more broiler were installed. How would your answer change if $p = 6$?

(c) Harder. Suppose that broilers cost r, measured on an hourly basis. (You will need to assume $r > 4p$.) Use derivatives and the chain rule to show how an increase in the price of broilers affects the number of workers hired. Note that you will have a system of two equations to deal with, but you should not have to solve the system simultaneously to solve this problem. If $p = 2.50$, does BK hire more or fewer works when the price of broilers rises?

4. There are 100 consumers, each with utility function

$$U(x, D) = \sqrt{x} - 2D + (m - px)$$

There is one firm, but it behaves like a perfect competitor and has production function

$$f(L, D) = L^{0.4} D^{0.4}$$

The factor prices of L and D are both 1.

(a) What is the equilibrium price and quantity? Note: each consumer ignores that increased consumption of x will slightly increase D. Assume m is sufficiently large that $m > px$ at the optimum.

(b) What is the social marginal cost of D?

(c) What is the efficient price of x and what is the efficient quantity?