Notes, Baumol Chapter 15

(1) $y_t = y_{pt} + y_{it}$ total R+D = price-sensitive R+D + price-insensitive R+D

(2) $g_{t+1} = s + b y_t$ growth rate in other industries = constant + increasing in R+D

(3) $P_{t+1} = P_t + v g_{t+1} \cdot P_t$ price of R+D rises in g - more R+D in t will lead to higher price of R+D in t+1.

(5) $y_{it+1} = h y_t$ as R+D in t increases, price insensitive R+D in t+1 just increases proportionately

$y_{pt+1} = a y_t - E \frac{P_{t+1} - P_t}{P_t} a y_t$ price sensitive R+D is also based on a proportionate target but is sensitive to changes in price based on a "bastard intertemporal demand elasticity."

$y_{pt+1} = a y_t - E v g_{t+1} a y_t$ subbing (3) into above

$y_{pt+1} = a y_t - k(s + b y_t) a y_t$ let $Ev = k$ and sub (2) into above

$y_{pt+1} = (a - aks) y_t - abk y_t^2$ rearranging

(12) $y_{t+1} = y_{pt+1} + y_{it+1} = (a + h - aks) y_t - abk y_t^2$ feedback relationship

(13) $y_{t+1} = y_t \Rightarrow y = 0$ or $y = \dfrac{a + h - aks - 1}{abk}$ steady states

(14) $\dfrac{dy_{t+1}}{dy_t} = a + h - aks - 2abk y_t$

(15) $\left.\dfrac{dy_{t+1}}{dy_t}\right|_{y_t = 0} = a + h - aks$

(16) $\left.\dfrac{dy_{t+1}}{dy_t}\right|_{y_t = \frac{a+h-aks-1}{abk}} = 2 - a - h + aks$

$\Big\}$ stability conditions for equilibria

$a < 1$, $h < 1$ because they are target shares of R+D

s is small because it's the rate of growth in the absense of R+D

$b < 1$ to keep growth rates in reasonable bounds

assume $k < 1$ / or not too much more / because plausible $E < 2$ and plausible v "small". This assumption seems much less certain to me.

So $y=0$, $\left.\frac{dy_{t+1}}{dy_t}\right|_{y=0} = a+h-aks > 0$ but < 1 unless h is large enough

$\Rightarrow$ stable unless h large

so in steady state, R+D declines to 0 unless there is a good deal of independent, price-insensitive innovation.

At $y = \frac{a+h-aks-1}{abk}$, $\frac{dy_{t+1}}{dy_t} = 2-a-h+aks > 0$ and > 1 unless h large

so for the small h case, this is a negative, unstable equilibrium and not meaningful

for the large h case, this could be stable and positive