ECON 201, Prof. Hogendorn: Problem Set #1

1. Suppose an economy has a production possibilities frontier characterized by the equation
   \[ X^2 + 4Y^2 = 100 \]
   (a) What is the most \( X \) that can be produced? The most \( Y \)?
   (b) Graph the frontier (you will have to find at least three points on the frontier to do this).
   (c) What is the formula for the opportunity cost of \( X \) in terms of \( Y \) in this economy? (You will need to solve for \( Y \) as a function of \( X \) and take the derivative, or else use differentials.)

2. Suppose you have an income of $40 to spend on two commodities. Commodity \( X \) costs $10 per unit and commodity \( Y \) costs $5 per unit.
   (a) Write down your budget constraint. If you spent all your income on \( X \), how much could you buy? If you spent all your income on \( Y \), how much could you buy? Graph your budget constraint. What is its slope?
   (b) Suppose the price of \( X \) falls to $5 while everything else stays the same. Write down your new budget constraint. What is its slope? Graph your new budget constraint on the same graph as (a).
   (c) Suppose your income falls to $30, but the prices of \( X \) and \( Y \) remain at $5. Write down your new budget constraint. What is its slope? Graph your new budget constraint on the same graph as (a) and (b).
   (d) On your graph, shade in the area representing commodity bundles that you can afford with the budget in (c) but could not afford to buy with (a). Shade in the area representing commodity bundles that you could afford with the budget in (a) but cannot afford with the budget in (c).

3. Suppose the economy in Problem 1 opens up to trade. In the world market, the price of \( X \) is $1 and the price of \( Y \) is also $1.
   (a) Let the total amount of wealth in the economy be \( W \). Write down the economy’s budget constraint.
   (b) Now consider that \( W \) is just the total value of all \( X \) and \( Y \) produced by the economy. Given that both \( X \) and \( Y \) have a price of $1 and given the PPF, what combination of \( X \) and \( Y \) produces the maximum \( W \)?
   (c) Graph the PPF and budget constraint at this maximum \( W \), and show the possible bundles of \((X, Y)\) that the economy could consume.